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Sherman Transformations for Functions on the Sphere

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Introduction

Let $S=S^d$ be the unit sphere in $\boldsymbol{R}^{d+1}$. It is well-known that functions and functionals f on S can be developed in the series of the spherical harmonics; $f=\sum_{n=0}^{\infty}f_n$, where f_n are spherical harmonic functions of degree n in $(d+1)$-dimensions. Certain function spaces or functional spaces on the sphere S can be characterized by the behavior of the sequence $\{\|f_n\|_2\}_{n=0,1,2,\cdots}$ (Lemma 1.1). On the other hand, T. O. Sherman [7] introduced the two transformations $f\to Ff(b, n)$ and $f\to F_*f(b, n)$ and studied the developments of functions or functionals on S using them.

In this paper we propose to replace his transformation F_* by a slightly different tranformation $F_\sharp$:

$$F_\sharp f(b, n)=F_*f_n(b, n) \quad \text{for} \quad f=\sum_{n=0}^{\infty}f_n .$$

Though $F_*f(b, n)$ is well-defined only for some differentiable functions, $F_\sharp f(b, n)$ can be defined for more general functions and functionals. And in the results of Sherman [7] we can replace $F_*f(b, n)$ by $F_\sharp f(b, n)$.

Here $Ff(b, n)$ and $F_\sharp f(b, n)$ are polynomials on the "equator" $B=\{s\in S;\ s\cdot a=0\}$, where $a=(0, 0, \cdots, 1)\in S$ denotes the "north pole".

The two transformations F and $F_\sharp$ define the mappings:

$$F: f \longrightarrow F(f)=\{Ff(\ ,n)\}_{n=0,1\cdots}\in \prod P_n(B)$$
$$F_\sharp: f \longrightarrow F_\sharp(f)=\{F_\sharp f(\ ,n)\}_{n=0,1\cdots}\in \prod P_n(B) ,$$

where $P_n(B)=\{g;$ a polynomial on B of degree at most $n\}$ and $\prod P_n(B)$ is the direct product of $P_n(B)$ $(n=0, 1, 2, \cdots)$. We call F the *Sherman transformation* and $F_\sharp$ the *modified Sherman transformation* respectively. Furthermore, we can consider that F and $F_\sharp$ are dual to each other in

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