TOKYO J. MATH.
VOL. 8, NO. 2, 1985

Lipschitz Classes of Periodic Stochastic Processes and Fourier Series

Tatsuo KAWATA

Keio University

Introduction

Let $X(t, \omega)$ be a complex valued stochastic process on a complete probability space (Ω, F, P), $t \in R^1$, $\omega \in \Omega$. Suppose throughout that $X(t, \omega)$ is measurable $L \times F$ on $R^1 \times \Omega$, L being the class of Lebesgue measurable sets on R^1. Assume also that $X(t, \omega)$ is an L^r-process, namely $X(t, \omega) \in L^r(\Omega)$ for each $t \in R^1$, $1 \leqq r < \infty$ and that $X(t, \omega)$ is 2π-periodic in the sense that

$$E|X(t+2\pi, \omega) - X(t, \omega)| = 0 \ , \tag{0.1}$$

for each $t \in R^1$. For an L^2-process $X(t, \omega)$, (0.1) is equivalent to $E|X(t+2\pi, \omega) - X(t, \omega)|^2 = 0$ which, as we easily see, is equivalent also to the condition that the covariance function $\rho(u, v)$ of $X(t, \omega)$ is 2π-periodic with respect to each of u and v.

Write

$$\begin{aligned} &\|X(t, \cdot)\|_r = \|X(t, \omega)\|_r = [E|X(t, \omega)|^r]^{1/r} \ , \\ &\|X(\cdot, \cdot)\|_{s,r} = \|X(t, \omega)\|_{s,r} = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} \Big\|X(t, \cdot)\Big\|_r^s dt\right]^{1/s} . \end{aligned} \tag{0.2}$$

The class of $X(t, \omega)$ for which $\|X(\cdot, \cdot)\|_{s,r} < \infty$ for some $1 \leqq r < \infty$, $1 \leqq s \leqq \infty$ is denoted by $L^{s,r} = L^{s,r}(T \times \Omega)$, $T = [-\pi, \pi]$.

Write, for a positive integer p, the p-th difference of $X(t, \omega)$ with increment h of t, by

$$\Delta_h^{(p)} X(t, \omega) = \sum_{k=0}^{p} (-1)^{p-k} \binom{p}{k} X(t+kh, \omega) \tag{0.3}$$

and define, for $\delta > 0$,

Received May 24, 1984
Revised November 1, 1984