TOKYO J. MATH.
VOL. 9, NO. 1, 1986

A Characterization of Cyclical Monotonicity by the Gâteaux Derivative

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(Communicated by K. Ogiue)

Introduction

Let X be a real Banach space and X' be its dual space. In this paper, we characterize the (maximal) cyclical monotonicity of a w^*-Gâteaux differentiable (nonlinear) operator: $X \to X'$, by means of the Gâteaux derivative. Our result is a nonlinear version of the well-known proposition; A linear and densely defined maximal monotone operator in a Hilbert space is cyclically monotone if and only if it is self-adjoint.

We give an equivalent condition for a w^*-Gâteaux differentiable operator from X to X' to be cyclically monotone, under some assumptions. Furthermore we give sufficient conditions for a (w-)Gâteaux differentiable operator in a Hilbert space to be maximal cyclically monotone. For instance, our Corollary 1 says that an operator A in a Hilbert space is maximal cyclically monotone, if $\overline{\delta A(x)}$, the minimal closed extension of the Gâteaux derivative of A at x, is positive self-adjoint for each x in the domain of A, under a suitable assumption.

§1. Preliminaries.

Throughout this paper we use the following notations and definitions.

X denotes a real Banach space with norm $\| \ \|$, and X' denotes its dual space. We denote by (x, f) the pairing between $x \in X$ and $f \in X'$. Especially if X is a real Hilbert space, $(\, , \,)$ is the inner product and we use the notation H instead of X.

For a subset S of X, $\bar{S}$ denotes the closure of S in X.

Let A be an operator from X to X'. $D(A)$ denotes the domain of A and $R(A)$ denotes the range of A. We denote the minimal closed extension of A by $\bar{A}$.

Received January 10, 1985