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Periodic Solutions on a Convex Energy Surface of a Hamiltonian System

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Introduction

Let $p=(p_1, \cdots, p_n)$, $q=(q_1, \cdots, q_n)$ be points of $\boldsymbol{R}^n$ and write $z=(p, q)\in\boldsymbol{R}^{2n}$. We consider a Hamiltonian system of $H\in C^2(\boldsymbol{R}^{2n}, \boldsymbol{R})$

$$(H) \qquad \dot{p}=-H_q\ ,\quad \dot{q}=H_p$$

or equivalently

$$(H) \qquad \dot{z}=JH'(z)\ ,\quad J=\begin{pmatrix}0 & -I\\ I & 0\end{pmatrix}\ ,$$

with I being the identity in $\boldsymbol{R}^n$.

On any compact energy surface for classical Hamiltonian, that is, $H=$"*kinetic energy*"+"*potential*", we have at least one periodic solution of (H) [6] [5].

For any star-shaped energy surface, there exists at least one periodic solution of (H) on it [7].

For a convex energy surface, Ekeland and Lasry [3] found n periodic solutions on it and Ambrosetti-Mancini [2] extended it to the following.

We define $[s]_+=[s]_-=s$ for $s\in\boldsymbol{Z}$ and $[s]_-=j$, $[s]_+=j+1$ for $s\in(j, j+1)$ with $j\in\boldsymbol{Z}$.

THEOREM 1. *Let C be a compact strictly convex subset of $\boldsymbol{R}^n$ with C^2 boundary S. For some $h\in\boldsymbol{R}$, $H^{-1}(h)=S$ and $H'(z)\neq 0$ for any $z\in S$.*

Assume further that there exist r, $R\in\boldsymbol{R}^+$ and $k\in\boldsymbol{Z}$, $2\leqq k\leqq n$, with

$$(0.1) \qquad R<\sqrt{k}\,r$$

such that

$$(0.2) \qquad rB\subset C\subset RB\ ,$$

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