TOKYO J. MATH.
VOL. 9, NO. 1, 1986

An Intermediate Value Theorem in Neighbourhood Spaces

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(Communicated by K. Kojima)

Introduction

In [6], the intermediate value theorem for fuzzy spaces was proved. These spaces were considered as an alternative of topological spaces. Hazy spaces were devised in [2] as an extension for fuzzy spaces. Neighbourhood spaces were considered in [3] as a generalization of hazy spaces. In this note an intermediate value theorem for neighbourhood maps on any connected neighbourhood space with its values on the standard neighbourhood space is given.

§1. Preliminaries.

We fix our terminology as in the following, for the detail of which one can see [3, 4].

DEFINITION. A *hazy space* is a pair (X, τ), where τ, the haze, is a reflexive and symmetric relation from the non-empty set X to its set of all subsets, Sub X. That is, $\tau \subseteq X \times \mathrm{Sub}\, X$, with for all $x, y \in X$

(i) $x \in \tau(x) = \bigcup_{(x,A) \in \tau} A$

(ii) $x \in \tau(y)$ iff $y \in \tau(x)$.

$\tau(x)$ is called the *neighbourhood* (abbr. to *nbd*) of x.

The set $\boldsymbol{Z}$ of all integers with standard haze,

$$\upsilon = \{(k, \{k, k-1\}), (k, \{k, k+1\}); k \in \boldsymbol{Z}\}$$

plays a role in hazy spaces corresponding to that of the Euclidean 1-dimensional space $\boldsymbol{R}$ in topological spaces. The observation that the nbds $\tau(x)$ play a much more prominent role than the subsets A was the motivation to develop the neighbourhood spaces in [3]. Structures of this kind were also introduced in [1] as neighbourhood systems.

Received October 24, 1984