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On the Decay of Correlation for Piecewise Monotonic Mappings II

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Introduction

In this paper, we will consider a class of piecewise linear transformations defined on the unit interval [0, 1]. We will show that under some suitable conditions the transformations belonging to this class exhibit mixing properties, and we derive estimates for the decay rate of correlation for them. Specifically, we will prove:

THEOREM 0-1. *Let F be a transformation on the unit interval* [0, 1] *satisfying conditions* i), ii), iii) *given in* §1. *Suppose that the infimum of the lower Lyapunov number is positive and the second Fredholm eigenvalue η is less than* 1. *Then F has a unique invariant probability measure μ absolutely continuous with respect to the Lebesgue measure on* [0, 1] *and the dynamical system* ([0, 1], μ, F) *is mixing, and the following estimate for the decay rate of correlation holds for any pair of functions $f\in BV$ and $g\in L^1$*:

$$\lim_{n\to\infty}(\eta+\varepsilon)^{-n}\left\{\int f(x)g(F^{(n)}(x))d\mu-\int fd\mu\int gd\mu\right\}=0\ , \tag{0.1}$$

for any $\varepsilon>0$.

This result extends the results obtained by the author in [8], and we will discuss in [9] some further results for more general cases. Some related topics have appeared in [2], [6], [10], [13] and [14]. Precise definitions of the lower Lyapunov number ξ and the second Fredholm eigenvalue η will be stated in §1.

Certain critical phenomena appear as $\xi\downarrow 0$, which indicates that the state of the system approaches the so-called window state. Concerning window states, we refer the readers to [3]. For the case where $\xi<0$,

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