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Some Skew Product Transformations Associated with Continued Fractions and Their Invariant Measures

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Introduction

In this paper we discuss the following number theoretical transformations defined on $[0, 1)\times[0, 1)$

$$T_1;\ (\alpha, \beta)\longrightarrow\left(\frac{1}{\alpha}-\left[\frac{1}{\alpha}\right], \frac{\beta}{\alpha}-\left[\frac{\beta}{\alpha}\right]\right)$$

and

$$T_2;\ (\alpha, \beta)\longrightarrow\left(\frac{1}{\alpha}-\left[\frac{1}{\alpha}\right], -\left[-\frac{\beta}{\alpha}\right]-\frac{\beta}{\alpha}\right).$$

These transformations T_1 and T_2 which can be found in [1] are examples of the so-called skew product transformations associated with the continued fraction transformation S; $\alpha\rightarrow(1/\alpha)-[1/\alpha]$. These transformations induce the following expansions, respectively (see §1 and §3 for details);

1) $\beta=\sum_{k=1}^{\infty}|\theta(k-1)|\cdot b(k)$

and

2) $\beta=\sum_{k=1}^{\infty}\theta(k-1)\cdot b'(k)$

where $\theta(n)=q_n\alpha-p_n$.

Therefore, the transformations T_1 and T_2 give the algorithms which will yield the approximations of the real number β by means of the set of all translates $\{n\alpha\}$ of an irrational number α.

In this paper we discuss the ergodic properties of the transformations T_1 and T_2. And we shall elaborate on number theoretical applica-

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