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An Immersion of an n-dimensional Real Space Form into an n-dimensional Complex Space Form

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Introduction

After the famous theorem of Hilbert "There exists no isometric immersion of a hyperbolic plane $H^2(-1)$ into a 3-dimensional Euclidean space." and his conjecture "There exists no isometric immersion of an n-dimensional hyperbolic space $H^n(-1)$ into a $(2n-1)$-dimensional Euclidean space." ([5]), we have studied the problem "Can an n-dimensional hyperbolic space $H^n(-1)$ be isometrically immersed in a Euclidean space R^N?" W. Henke ([4]) constructed an isometric immersion $H^n(-1) \to R^{4n-3}$. But few facts have been known beyond them.

In this paper, we get an example of a local immersion of $H^n(-1)$ into an n-dimensional complex Euclidean space C^n, as a totally real submanifold. Moreover we can determine the immersion of a real space form $M^n(c)$ into a complex space form $\tilde{M}^n(4\tilde{c})$ for $c<\tilde{c}$ as a totally real submanifold with a certain condition about a mean curvature vector (§1). This is a natural extention of the Ejiri's Theorem in [2] and contains an example of Vranceanu [6].

We remark that this immersion cannot be extended globally.

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§1. Chen submanifolds.

Let M be a submanifold immersed in $\tilde{M}$. We denote by $\langle\ ,\ \rangle$ the Riemannian metrics on M and $\tilde{M}$. Let σ and h be the second fundamental form and the mean curvature vector of the immersion, respectively.

DEFINITION 1.1. A submanifold M immersed in $\tilde{M}$ is called a Chen submanifold if it satisfies the condition

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