TOKYO J. MATH.
VOL. 9, NO. 1, 1986

Normal Forms for Certain Singularities of Smooth Map-Germs

Fumio ICHIKAWA

Tokyo Metropolitan University
(Communicated by K. Ogiue)

In the theory of singularity of smooth mapping, finite determinacy has been studied by many authors [6]. In [4], J. Mather gave a complete characterization of finite determinacy, but in general it is very difficult to check whether a given map-germ $f: (R^n, 0) \to (R^p, 0)$ is finitely determined or not except for stable singularities or the case $p=1$. In this paper we give some classification of smooth mappings $f: (R^n, 0) \to (R^2, 0)$ by an elementary method.

In §1 we recall J. Mather's theorem on finite determinacy.

In §2 we prove what we call Normal Form Theorem (Theorem 2.1, Theorem 2.5 and Theorem 2.7). In Theorem 2.1 we give normal forms of function-germs. As its immediate corollaries we obtain the Morse lemma (Example 2.3) and the splitting lemma for functions (Example 2.4). These corollaries are well-known and have nothing new, however from these examples show how convenient and efficient it will be if we generalize Theorem 2.1 to the case of map-germs. This is what we have done. (Theorem 2.5, Theorem 2.6).

In §3 we prove the Splitting Lemmas for map-germs of corank 1 (Theorem 3.2 and Theorem 3.3) using the normal forms obtained in §2.

In §4 as an application of our normal forms and splitting lemmas, we classify finitely determined map-germs of R^n into R^2 of corank 1 whose 3-jets are non-trivial. An estimation of order of their determinacy is given as well. From the splitting lemmas develloped in §3, the classification and the estimation of order of determinacy of these map-germs are reduced to those of map-germs of plane to plane. Then they are carried out in a rather elementary way.

§1. Preliminaries.

In this section we recall Mather's theorem. Let $\mathscr{E}_n$ be the ring of

Received September 26, 1984