TOKYO J. MATH.
VOL. 9, NO. 1, 1986

Lower Bounds for the Unknotting Numbers of Certain Iterated Torus Knots

Makoto YAMAMOTO[1]

Waseda University

Introduction

In [5], Milnor asked if the unknotting number of an algebraic knot is equal to its genus, where an algebraic knot is the one which occurs as an isolated singularity of a complex plane curve (see Problem 4 of Gordon [2] also). It is known that an algebraic knot is an iterated torus knot and that all torus knots are algebraic. It is a consequence from a result of Murasugi [6] that the problem of Milnor is affirmative for the $(2, m)$-torus knot. It is not hard to see that the unknotting number of an algebraic knot is at most its genus. It is more difficult to give lower bounds for the unknotting number. Weintraub [7] showed that the unknotting number of $(m-1, m)$-torus knot is at least $(m^2-5)/4$ if m is odd, and $(m^2-4)/4$ if m is even. Yamamoto [9] showed that the unknotting number of $(l, 2kl\pm1)$-torus knot is at least $(k(l^2-1)-2)/2$ if l is odd, and $(kl^2-2)/2$ if l is even. In this note we give lower bounds for the unknotting numbers of certain iterated torus knots as follows (see §1 for definitions and notations);

THEOREM. *The unknotting number of the* $([l_1, 2kl_1-1], [l_2, 2kl_1l_2+(l_1-1)l_2-1], \cdots, [l_q, 2kl_1\cdots l_q+(l_{q-1}-1)l_q-1])$*-iterated torus knot is at least* $(k((l_1\cdots l_q)^2-1)-2)/2$ *if* $l_1\cdots l_q$ *is a power of an odd prime, and* $(k(l_1\cdots l_q)^2-2)/2$ *if* $l_1\cdots l_q$ *is even, where* k *is a positive integer and* $q, l_1, \cdots, l_q$ *are integers greater than one.*

With our notation, the $([l_1, m_1], \cdots, [l_q, m_q])$-iterated torus knot is algebraic if and only if $m_il_{i+1}<m_{i+1}$ for all $i=1, \cdots, q-1$ by Brauner [1] (cf. Lê [4]). Then we note that the $([l_1, 2kl_1-1], [l_2, 2kl_1l_2+(l_1-1)l_2-1])$-

Received February 7, 1985
Revised May 22, 1985

[1] Supported in part by the Grand-in-Aid for Science Research (No. 59740054), Ministry of Education, Japan.