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Superficial Saturation

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Introduction

Let A be a Cohen-Macaulay semi-local ring of dimension d, I an ideal of definition of A and $P(I, t)=\sum_{n>0}^{\infty}\lambda(I^n/I^{n+1})t^n$ the associated Poincaré series where $\lambda(\ )$ denotes the length of A-module. Then $P(I, t)$ is of the form $e_0(1-t)^{-d}-e_1(1-t)^{1-d}+\cdots+(-1)^{d-1}e_{d-1}(1-t)^{-1}+(-1)^d e_d^{(0)}+(-1)^d e_d^{(1)}t+\cdots+(-1)^d e_d^{(r)}t^r$. The coefficients e_k $(0\leqq k\leqq d)$ are the so called normalized Hilbert-Samuel coefficients of I with $e_d=e_d^{(0)}+e_d^{(1)}+\cdots+e_d^{(r)}$. Since $\sum_{i=0}^{k}\binom{d+i-1}{i}=\binom{k+d}{d}$, the Hilbert-Samuel function $\lambda(A/I^{n+1})$ of I equals $e_0\binom{n+d}{d}-e_1\binom{n+d-1}{d-1}+\cdots+(-1)^{d-1}e_{d-1}\binom{n+1}{1}+(-1)^d e_d$ for each $n>r$. We say that $e_0(1-t)^{-d}+\cdots+(-1)^{d-1}e_{d-1}(1-t)^{-1}$ and $(-1)^d(e_d^{(0)}+e_d^{(1)}t+\cdots+e_d^{(r)}t^r)$ are respectively the principal part and the polynomial part of the Poincaré series. In this paper we assume that A/P is infinite for each maximal ideal P, which guarantees the existence of superficial elements. A superficial element x of I is said to be stable if $I^n\colon x=I^{n-1}$ for all $n>1$. We say that a sequence of d elements $x_1, \cdots, x_d$ of I is an I-superficial (resp. a stable I-superficial) sequence, if x_k mod. $(x_1, \cdots, x_{k-1})$ is a (resp. stable) superficial element of $I/(x_1, \cdots, x_{k-1})$ for each k $(1\leqq k\leqq d)$. For an I-superficial sequence $x_1, \cdots, x_d$, there exists $m>0$ such that $(x_1, \cdots, x_d)I^m=I^{m+1}$. We evaluate m in section 1.

Now in case $d=1$, I is said to be stable if it satisfies one of the following equivalent conditions.

(i) $\lambda(A/I^n)$ is a polynomial in n for all $n>0$.

(ii) $xI=I^2$ for some x in I.

(iii) $P(I, t)$ is of the form $e_0(1-t)^{-1}-e_1$ (see [6]).

In the case of dimension $d>1$, the theory of stable ideals can be extended in two directions. One is about the ideals such that $(x_1, \cdots, x_d)I=I^2$ for some $x_1, \cdots, x_d$ in I. The other is about the ideals satisfying the above

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