TOKYO J. MATH.
VOL. 9, No. 1, 1986

On a Formula of Morita's Partition function $q(n)$

To the memory of Dr. Takehiko Miyata

Hiroshi NARUSE

Okayama University

(Communicated by Y. Kawada)

Introduction

It is well known that the number of conjugacy classes of $\mathfrak{sl}(2, \boldsymbol{C})$ in the Lie algebra of type $A_{n-1}=\mathfrak{sl}(n, \boldsymbol{C})$ is $p(n)-1$, where $p(n)$ is the number of partitions of n. Recently J. Morita [1] found that the number of conjugacy classes of $\mathfrak{sl}(2, \boldsymbol{C})$ in the Kac-Moody Lie algebra of type $A_{n-1}^{(1)}$ is finite and that this number is given by $q(n)-1$ where $q(n)$ is the function defined by (1), which we call Morita's partition function. But it is not easy to calculate $q(n)$ directly following the definition. In this note, using the convolution product, we give a formula of $q(n)$ (Theorem) which seems to have some significance in itself. We also give a combinatorial proof of this formula.

We would like to express great thanks to Professor Jun Morita for communicating this problem.

§1. Notations.

Let $\boldsymbol{Z}_+=\{1, 2, 3, \cdots\}$ be the set of positive integers. For $n \in \boldsymbol{Z}_+$, a partition of n is a sequence $\lambda=(\lambda_1, \lambda_2, \cdots, \lambda_r)$, where $\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_r$, $\lambda_i \in \boldsymbol{Z}_+$ and $\sum_i \lambda_i = n$. We write $\lambda \vdash n$ if λ is a partition of n. For a partition $\lambda=(\lambda_1, \lambda_2, \cdots, \lambda_r)$, we define a number $a(\lambda)$ by

$$a(\lambda)=G.C.D.(\lambda_1, \lambda_2, \cdots, \lambda_r)$$

the greatest common divisor.

We denote by $\mathfrak{Z}$ the set of functions from $\boldsymbol{Z}_+$ to the complex numbers $\boldsymbol{C}$. Let us denote by $f*g$ the convolution product of $f, g \in \mathfrak{Z}$, i.e.

$$f*g(n)=\sum_{d|n} f(d)g\left(\frac{n}{d}\right) .$$

Received October 1, 1984