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On the Quartic Residue Symbol of Totally Positive Quadratic Units

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Introduction

Let m be a square free positive integer and ε_m the fundamental unit of the quadratic field $\boldsymbol{Q}(\sqrt{m})$. If ε_m is totally positive, then we define the biquadratic symbol $(\varepsilon_m/p)_4$ for the rational prime number p with the condition,

$$(*) \qquad (-1/p)=(m/p)=(\varepsilon_m/p)=1 .$$

We refer to [3] for the definitions of the symbols (ε_m/p) and $(\varepsilon_m/p)_4$. Let K (resp. K') be the Galois extension over the rational number field $\boldsymbol{Q}$ generated by $\sqrt{-1}$ and $\sqrt[4]{\varepsilon_m}$ (resp. $\sqrt{-1}$ and $\sqrt{\varepsilon_m}$). Then the condition $(*)$ is equivalent to say that p splits completely in K'. Further the symbol $(\varepsilon_m/p)_4$ expresses the decompition law of this prime p between K and K'. Let T_m be the trace of ε_m over $\boldsymbol{Q}$ and denote by f_m (resp. e_m) the square free part of T_m+2 (resp. $m(T_m+2)$). Consider the following three quadratic fields;

$$(1) \qquad F=\boldsymbol{Q}(\sqrt{f_m}) , \qquad E=\boldsymbol{Q}(\sqrt{-e_m}) , \qquad k=\boldsymbol{Q}(\sqrt{-m}) .$$

Then K contains all these quadratic fields and is abelian over each of them. If the ideal class groups corresponding to K and K' in each field of (1) are determined explicitly, then we obtain three sorts of expressions of $(\varepsilon_m/p)_4$ in view of the representation of a power of p by the norm form of each quadratic field. In the present paper, we offer explicit expressions of this symbol for the integers m of following types:

$$(2) \qquad \begin{aligned} &m=qq': \quad q\equiv 5, 3 \bmod 8 , \quad q'\equiv 3 \bmod 4 , \quad (q/q')=-1 ; \\ &m=2q: \quad q\equiv 3 \bmod 8 ; \\ &m=q: \quad q\equiv 3, 7, 11 \bmod 16 . \quad (q, q': \text{prime numbers.}) \end{aligned}$$

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