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Decomposition and Inertia Groups in Z_p-Extensions

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Introduction

In this paper, we shall give a canonical description of the decomposition and inertia groups, in a Z_p-extension of a number field k, for any prime ideal of k; when this prime ideal divides p, and ramifies in the Z_p-extension, all the higher ramification groups are also described. This description gives immediately a numerical knowledge of the previous groups, as soon as the p-class group and the group of units of k are numerically known.

Of course, if K/k is any abelian extension, the law of decomposition of prime ideals of k is known if and only if the Artin group of K/k is given; but in practice we have the opposite situation: the extension K/k is specified by mean of some property (for instance K/k is a Z_p-extension...) and the problem is to determine its Artin group. The results obtained in [2] give a general method for this kind of problem, via the use of a logarithm function, Log, which induces a canonical isomorphism between the Galois group G of the compositum $\tilde{k}$ of all Z_p-extensions of k, and an explicit Z_p-module attached to k and depending (numerically) on ideal classes and units. It is well known that the decomposition group $G_{\mathfrak{q}}$ in $\tilde{k}/k$, of any prime ideal $\mathfrak{q}$ of k, is the closure in G of the image of $k^\times$ by the Hasse norm residue symbol $((\ , \tilde{k}/k)/\mathfrak{q})$; then it is sufficient to compute $\mathrm{Log}((a, \tilde{k}/k)/\mathfrak{q})$ for any $a \in k^\times$; we obtain an explicit formula for $\mathrm{Log}((a, \tilde{k}/k)/\mathfrak{q})$ which permits us to describe $G_{\mathfrak{q}}$ and its subgroups such as the inertia and higher ramification groups and to give some properties of jumps of ramification (§2). Finally, to illustrate this study, we consider (in §3) the case of imaginary quadratic fields and give all details for a numerical utilization.

Some basic tools used in this paper may be compared with some ones

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