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Examples of Simply Connected Compact Complex 3-folds II

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Introduction

This note is the continuation of [2]. In [2], the first named author has constructed a series of compact complex manifolds $\{M_n\}_{n=1,2,3,\dots}$ of dimension 3 which are non-algebraic and non-Kaehler with the properties: $\pi_1(M_n)=0$, $\pi_2(M_n)=\boldsymbol{Z}$, $b_3(M_n)=4n$, $\dim H^1(M_n, \mathcal{O})\geqq n$, and $\dim H^1(M_n, \Omega^1)\geqq n$. The present note consists of two sections, § 5, § 6. In section 5, we shall show how to describe differentiable structures of $\{M_n\}$ in terms of connected sums, using a result of C. T. C. Wall [4]. We note, in particular, that M_1 is diffeomorphic to the connected sum of twice $S^3\times S^3$ and $S^2\times S^4$; $M_1\approx 2(S^3\times S^3)\#_t S^2\times S^4$ and that M_2 is diffeomorphic to that of 4 times $S^3\times S^3$ and $\boldsymbol{P}^3$; $M_2\approx 4(S^3\times S^3)\#_t P^3$. Here $\#_t$ indicates the usual connected sum in the category of differentiable topology. In section 6, we shall calculate all of their Hodge invariants. We have $\dim H^1(M_n, \mathcal{O})=n$ and $\dim H^1(M_n, \Omega^1)=n+1$, while $H_1(M_n, \boldsymbol{Z})=0$ and $H_2(M_n, \boldsymbol{Z})=\boldsymbol{Z}$.

In the following, we shall use the notation in [2].

§ **5.** In this section, we shall study the differentiable structures of the compact complex manifolds of dimension 3 $\{M_n\}_{n=1,2,3,\dots}$, which were constructed in [2].

LEMMA 11.

(v)
$$H_q(M_n, \boldsymbol{Z})=\begin{cases}\boldsymbol{Z} & q\text{: even },\\ 0 & q=1,5 ,\\ \boldsymbol{Z}^{4n} & q=3 .\end{cases}$$

(vi) *Let l be a projective line in $\Sigma\subset\boldsymbol{P}^3$. Then, for any $n\geqq 1$, $l_n:=i_1(l)$ $(\subset M_1^{n-1}\subset M_n)$ represents a generator of $H_2(M_n, \boldsymbol{Z})$, where M_1^0 is understood to be M_1.*

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