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On a Bernoulli Property for Multi-dimensional Mappings with Finite Range Structure

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Introduction

In the previous paper [4], we considered ergodic properties of a mapping T defined on a bounded domain $X \subset R^d$ satisfying a "local Renyi's condition". The purpose of this paper is to prove that such a mapping T is weak Bernoulli if it admits a finite absolutely continuous invariant measure.

The mapping we consider is characterized by a certain type of partition $Q=\{X_a: a \in I\}$ of X and a finite number of subsets $U_0(=X)$, U_1, $\cdots$, U_N of X satisfying some special properties (see §1 for precise definitions). We shall call such a transformation T a multi-dimensional mapping with a finite range structure. If such a T satisfies the Renyi's condition, in addition, then it is known that T has a finite absolutely continuous invariant measure, and furthermore, under some additional conditions one can prove that Q is a weak Bernoulli partition ([9], [18]). On the other hand, when X is an interval of R^1, Ledrappier established in [6] the weak Bernoulli property for a transformation T having a similar characterization under some further hypothesis, such as the existence of a finite invariant measure with positive entropy, but without assuming that T satisfies the Renyi's condition (cf. [2]). The main ingredient of his proof, which is patterned after the work of Sinai [15] (cf. [16]) and Ratner [12], is the use of Rohlin's formula for proving the absolute continuity of some conditional measures.

In this paper we establish a sufficient condition for a multi-dimensional mapping with a finite range structure to have the weak Bernoulli property when they do not necessarily satisfy the Renyi's condition. We do need, however, to make several assumptions on the transformation; some of these assumptions seem to be essential, while the others are seen

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