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Periodic Solutions on a Convex Energy Surface of a Hamiltonian System II

A Quantitative Estimate for Theorem by A. Weinstein Concerning Normal Modes

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Introduction

Let $p, q \in \boldsymbol{R}^n$ and $H=H(p, q) \in C^1(\boldsymbol{R}^{2n}, \boldsymbol{R})$. We consider a Hamiltonian system

$$(H) \qquad \dot{p}=-H_q\,, \qquad \dot{q}=H_p\,,$$

or concisely

$$(H) \qquad \dot{z}=JH'(z)\,,$$

where $z=(p, q)$ and $J=\begin{pmatrix}0 & -I\\ I & 0\end{pmatrix}$ with I being the identity of $\boldsymbol{R}^n$.

In 1973, A. Weinstein [10] obtained an interesting

THEOREM 1. *Let* $H \in C^2(\boldsymbol{R}^{2n}, \boldsymbol{R})$, $H(0)=0$, $H'(0)=0$ *and* $H''(0)>0$. *Then for sufficiently small* $\varepsilon>0$, *there exist* n *distinct periodic solutions of* (H) *on* $H^{-1}(\varepsilon)$.

For small $\varepsilon>0$, $H^{-1}(\varepsilon)$ is a convex manifold (manifold which bounds a usual convex set) close to an ellipsoid.

For ellipsoids, that is, for the system describing harmonic oscillators, there are exactly n periodic solutions on any energy surface if the angular frequencies are independent over $\boldsymbol{Q}$.

In 1978, A. Weinstein [11] also found at least *one* periodic solution on any convex Hamiltonian energy surface. Although that was covered by P. Rabinowitz [9] which gave the result for star-shaped one, an estimate obtained in [11] was used in [6].

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