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Maslov's Quantization Conditions for the Bound States of the Hydrogen Atom

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Introduction

The purpose of this paper is to show that Maslov's quantization condition determines the eigenvalues of the Schrödinger operator of the hydrogen atom, the angular momentum operator and the Lenz operator, and also determines multiplicities of the eigenspaces for the hydrogen atom. Namely, we concern the eigenvalue problem of the following Schrödinger operator on $\boldsymbol{R}^3$:

$$(1) \qquad \hat{H}\left(x, \frac{h}{i}\frac{\partial}{\partial x}\right) = -\frac{h^2}{2}\Delta - \frac{1}{|x|}, \qquad |x| = \left(\sum_{k=1}^{3} x_k^2\right)^{1/2},$$

where h is a positive parameter and Δ is the 3-dimensional Laplacian.

Maslov [7], [8] introduces his index and the quantization condition for Lagrangian submanifolds and studies the "asymptotic solutions" of the eigenvalue problems in quantum mechanics (cf. [4]). On the contrary, *in our case*, we get exact values. Thus, as far as these systems are concerned, we need not to consider the operator theory to obtain the exact quantum mechanical conclusion. What we need is only classical mechanics, invariant Lagrangian submanifolds and Maslov's quantization conditions.

Associated with (1), we consider the following commuting system of operators $\{\hat{H}(x, (h/i)(\partial/\partial x)), \hat{l}_1(x, (h/i)(\partial/\partial x)), \hat{e}_1(x, (h/i)(\partial/\partial x))\}$, where $\hat{l}_1(x, (h/i)(\partial/x))$ and $\hat{e}_1(x, (h/i)(\partial/\partial x))$ are the angular momentum operator and the Lenz operator, respectively, defined by

$$(2) \qquad \hat{l}_1\left(x, \frac{h}{i}\frac{\partial}{\partial x}\right) = x_2\hat{p}_3 - x_3\hat{p}_2 ,$$

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