TOKYO J. MATH.
VOL. 9, NO. 2, 1986

Hilbert Transforms on One Parameter Groups of Operators

Shiro ISHIKAWA

Keio University

Introduction

In [1], M. Cotlar showed that M. Riesz's theorem could be extended to the case of a measure preserving flow as well as a real line or a circle. In this paper, more generally, we shall consider Hilbert transform on a one parameter group of operators on a complete locally convex space. For this, we define several terms and prepare some lemmas in what follows.

DEFINITION 1. Let $\boldsymbol{R}$ be a real field and let X be a complete locally convex space. Then $\{U_t; t\in \boldsymbol{R}\}$ is said to be a one parameter group of operators on X, if the following conditions are satisfied;

(i) U_t is a continuous linear operator on X for all $t\in\boldsymbol{R}$, and U_0 is an identity operator on X,

(ii) $U_tU_s=U_{t+s}$ for all $t, s\in\boldsymbol{R}$,

(iii) for any $t\in\boldsymbol{R}$ and any $x\in X$, $(U_{t+h}-U_t)x$ converges to 0 as $h\to 0$ in the topology of X (for short, in X).

DEFINITION 2. A continuous linear operator $H_{\varepsilon,N}(0<\varepsilon<N<\infty)$ on X is defined as follows;

$$H_{\varepsilon,N}x=\frac{1}{\pi}\int_{\varepsilon<|t|<N}\frac{U_tx}{t}dt \quad (x\in X)$$

(this integral can be well defined since a mapping $t\in R\to(U_tx)/t\in X$ is continuous on a compact set $\{t\in\boldsymbol{R}; \varepsilon\leqq|t|\leqq N\}$). Also, if $\lim_{\varepsilon\to 0+,N\to\infty}H_{\varepsilon,N}x$ exists in X, we denote it by Hx and call it a Hilbert transform of x. And the domain of H (i.e. $\{x\in X; Hx \text{ exists}\}$) is denoted by $D(H)$.

LEMMA 1. *Let X be a complete locally convex space and let $\{U_t; t\in\boldsymbol{R}\}$ be a one parameter group of operators on X. Let x be any element in X represented by*

Received May 7, 1985