TOKYO J. MATH.
VOL. 9, NO. 2, 1986

Artin's L-functions and Gassmann Equivalence

Kiyoshi NAGATA

Sophia University
(Communicated by Y. Kawada)

Introduction

R. Perlis, in [3], showed the following theorem.

Theorem. (Cassels & Fröhlich [5, p. 363 ex. 6.4])
Let L be a finite Galois extension of $\boldsymbol{Q}$, let $G=\mathrm{Gal}(L/\boldsymbol{Q})$, and let E and E' be subfields of L corresponding to the subgroups H and H' of G respectively. Then the following conditions are equivalent:
(1) H and H' are Gassmann equivalent in G.
(2) $\zeta_E(s)=\zeta_{E'}(s)$.
(3) The same primes p are ramified in E as in E', and for the non-ramified p the decomposition of p in E and E' is the same.

We shall extend this theorem in case that the base field is not necessarily $\boldsymbol{Q}$. In this case the condition (2) is not sufficient for (1). So we will replace ζ-function by Artin's L-functions for some characters.

T. Funakura, in [4], constructed the isomorphism Φ from the additive group $\mathrm{Ch}(\mathfrak{G})$ to $\mathfrak{L}$ where $\mathrm{Ch}(\mathfrak{G})$ is the character group of $\mathfrak{G}=G(\bar{\boldsymbol{Q}}/\boldsymbol{Q})$ and $\mathfrak{L}=\{L(s, \psi, E/\boldsymbol{Q}): E$ is a subfield of $\bar{\boldsymbol{Q}}$ corresponding to an open normal subgroup $\mathfrak{Y}$ s.t. $\psi\in\mathrm{Ch}(\mathfrak{G}/\mathfrak{Y})\}$. And he showed the equivalent relation between (1) and (2) as a corollary of his theorem. To show that Φ is monomorphic, he used the fact that the L-functions for the irreducible characters of $\mathrm{Gal}(E/\boldsymbol{Q})$ are multiplicativelly independent. We shall show the equivalence of (1) and (2) if the L-functions for the irreducible characters of $\mathrm{Gal}(E/k)$ are multiplicativelly independent.

§1. Preliminaries.

Now let k, K, K' be finite algebraic number fields where K, K' contain k. For a relative normal algebraic extension N over k which contains both K and K', we put $G=\mathrm{Gal}(N/k)$, $H=\mathrm{Gal}(N/K)$, $H'=\mathrm{Gal}(N/K')$. Then

Received June 4, 1985