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Quadratic Fields and Factors of Cyclotomic Polynomials

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Introduction

Let p be an odd prime and put $p^*=(-1)^{(p-1)/2}p$. The decomposition of the p-th cyclotomic polynomial $\Phi_p(X)$ in the quadratic field $\boldsymbol{Q}(\sqrt{p^*})$ has been early investigated in Dirichlet [2] and also in Staudt [9] and Lerch [6], the latter considered the decomposition of $\Phi_{|D|}(X)$ in $\boldsymbol{Q}(\sqrt{D})$ for a quadratic discriminant D. Later, Petersson [7], and recently, Williams [12], [13] have also investigated such a decomposition. Let m be the product of r distinct odd primes p_j $(1\leqq j\leqq r)$. In this paper, we consider the decomposition of $\Phi_m(X)$ in the field generated by all $\sqrt{p_j^*}$ over the field $\boldsymbol{Q}$ of rational numbers. Let $\Psi_m(X)$ be an irreducible factor of this decomposition. We derive the formulas for $\Psi_m(1)$, $\Psi_m(1^{1/4})$ and $\Psi_m(1^{1/8})$. Those formulas and our method are not found in any other paper. Our formulas are useful to obtain various results for quadratic fields. We derive certain congruences for the class numbers of imaginary quadratic fields. Those congruences essentially contain lots of results obtained by Lerch [6], Pitzer [8] and Berndt [1] through various methods different from ours. Also we derive some results concerning the class numbers and the norms of the fundamental units of real quadratic fields.

§1. Notation.

For a quadratic discriminant d, $h(d)$ (respectively $\hat{h}(d)$) denote the wide (resp. narrow) class number of the quadratic field $\boldsymbol{Q}(\sqrt{d})$. For $d>0$, ε_d (>1) denotes the fundamental unit of $\boldsymbol{Q}(\sqrt{d})$, and $\hat{\varepsilon}_d$ (>1) denotes the least totally positive unit of $\boldsymbol{Q}(\sqrt{d})$. For $d<0$, we have $h(d)=\hat{h}(d)$. For $d>0$, we have $h(d)=\hat{h}(d)$, $\varepsilon_d^2=\hat{\varepsilon}_d$ (resp. $2h(d)=\hat{h}(d)$, $\varepsilon_d=\hat{\varepsilon}_d$), if $N(\varepsilon_d)=-1$ (resp. $+1$), where N denotes the absolute norm. Let χ_d denote the Kronecker symbol of $\boldsymbol{Q}(\sqrt{d})$. For a prime q, q^* denotes a

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