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On the Curvature of Affine Homogeneous Convex Domains

Tadashi TSUJI

Mie University

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Introduction

Let Ω be a convex domain not containing any affine line in a real number space, and $G(\Omega)$ the group of all affine automorphisms of Ω. If the group $G(\Omega)$ acts on Ω transitively, then Ω is said to be homogeneous. For a convex domain Ω in the n-dimensional real number space, we denote by $D(\Omega)$ the tube domain over Ω in the n-dimensional complex number space. Then the tube domain $D(\Omega)$ is holomorphically equivalent to a bounded domain. Therefore, $D(\Omega)$ admits the Bergman metric. By restricting the Bergman metric of $D(\Omega)$ to Ω, we have a $G(\Omega)$-invariant Riemannian metric g_Ω on Ω, which is called the canonical metric of Ω ([13], [8]). Concerning the Bergman metrics of homogeneous bounded domains, D'Atri and Miatello ([2]) proved that a homogeneous bounded domain is symmetric if and only if the sectional curvature is non-positive. On the other hand, it is known that there exist non-symmetric homogeneous convex cones of non-positive sectional curvature in the canonical metric ([7]). The purpose of the present note is to determine homogeneous convex domains of non-positive sectional curvature among homogeneous convex domains of certain types or of low rank (Theorems 2.3, 3.3, 3.4, and 4.3).

The proofs for our results are carried out by calculations on T-algebras due to Vinberg ([13], [14]). The same notations and definitions as those in [6]–[10] will be employed.

§1. Preliminaries.

In this section, we fix notation and recall fundamental results on homogeneous convex domains and T-algebras mainly due to E. B. Vinberg ([13], [14]).

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