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Local Isometric Embedding of Two Dimensional Riemannian Manifolds into R^3 with Nonpositive Gaussian Curvature

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Introduction

As it is well known, the problem of C^∞ local isometric embedding of a two dimensional Riemannian manifold into R^3 is a problem whether C^∞ functions $x(u, v)$, $y(u, v)$, $z(u, v)$ which satisfy

$$dx^2+dy^2+dz^2=Edu^2+2Fdudv+Gdv^2 \tag{0.1}$$

exist in a neighborhood of a point, say $(u, v)=0$, when the first fundamental form $Edu^2+2Fdudv+Gdv^2$ is given. The results already known are as follows. Let K be the Gaussian curvature of the two dimensional manifold, then the classical result is that the problem is affirmatively answered if $K\neq 0$ at $(u, v)=0$, and a recent interesting result due to Lin [4] is that it is also affirmative if $K=0$, $\operatorname{grad} K\neq 0$ at $(u, v)=0$. Now a natural question arises. Namely, is it affirmative when

$$K=\operatorname{grad} K=0 \quad \text{at} \quad (u, v)=0 \tag{0.2}$$

and one of the following conditions holds:

(i) $\operatorname{Hess} K(0, 0)>0$,
(ii) $\operatorname{Hess} K(0, 0)<0$,
(iii) $\operatorname{Hess} K(0, 0)$ has two eigenvalues with opposite signs?

Hereafter, for simplicity, we refer the case with conditions (0.2) and (i) (resp. (ii) and resp. (iii)) by (i) (resp. (ii) and resp. (iii)).

Then what we have obtained is the following.

THEOREM. *The problem of C^∞ local isometric embedding is also affirmative in the case* (ii). (Consult §4 for the related results.)

We note we can replace the equation (0.1) by a second order Monge-

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