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Minimal Immersions of Kaehler Manifolds into Complex Space Forms

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Introduction

Let $N^m(\tilde{c})$ be an m-dimensional complex space form of constant holomorphic sectional curvature $\tilde{c}$, and let (M^n, g) be an n-dimensional Kaehler manifold. It is well known that holomorphic isometric immersions of Kaehler manifolds into Kaehler manifolds are minimal immersions. We consider the following problem: Is an isometric minimal immersion $f: (M^n, g) \to N^m(\tilde{c})$ a holomorphic or anti-holomorphic immersion? However, it is not true in general. For example, if we take $M^1 = \boldsymbol{RH}^2(\tilde{c}/4)$ (2-dimensional real hyperbolic space of constant curvature $\tilde{c}/4$, $\tilde{c}<0$) or $M^1 = S^2(\tilde{c}/4)$ (2-dimensional sphere of constant curvature $\tilde{c}/4$, $\tilde{c}>0$), then we obtain totally real isometric minimal immersions as follows:

$$(1) \qquad M^1 = \boldsymbol{RH}^2(\tilde{c}/4) \xrightarrow{\text{totally real, totally geodesic}} \boldsymbol{CH}^m(\tilde{c}) ,$$

where $\boldsymbol{CH}^m(\tilde{c})$ is an m-dimensional complex hyperbolic space of constant holomorphic sectional curvature $\tilde{c}$.

$$(2) \qquad M^1 = S^2(\tilde{c}/4) \xrightarrow{\text{natural covering}} \boldsymbol{RP}^2(\tilde{c}/4) \xrightarrow{\text{totally real, totally geodesic}} \boldsymbol{CP}^m(\tilde{c}) ,$$

where $\boldsymbol{CP}^m(\tilde{c})$ is an m-dimensional complex projective space of constant holomorphic sectional curvature $\tilde{c}$ and $\boldsymbol{RP}^2(\tilde{c}/4)$ is a 2-dimensional real projective space of constant curvature $\tilde{c}/4$.

In Part I, we prove the following

THEOREM 1. *Let $\boldsymbol{CH}^m(\tilde{c})$ be an m-dimensional complex hyperbolic space of constant holomorphic sectional curvature $\tilde{c}$ ($\tilde{c}<0$), and let (M^n, g) be an n-dimensional Kaehler manifold such that* $\dim_{\boldsymbol{C}} M = n \geqq 2$. *Then,*

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