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Isogenies between Algebraic Surfaces with Geometric Genus One

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The classical theory of the Albanese variety provides a geometric interpretation of first cohomology groups of complex projective varieties in the following way: a variety X and its Albanese variety $\mathrm{Alb}(X)$ have isomorphic first cohomology groups, and there is a mapping $X \to \mathrm{Alb}(X)$ inducing this isomorphism, the formation of which is functorial for maps between complex varieties.

This note develops the beginnings of an analogous theory for second cohomology groups of algebraic surfaces with geometric genus one. Specifically, we show that to any such surface is associated a so-called K3 surface, whose transcendental second cohomology is isomorphic to that of the original surface. This isomorphism is not in general given by a mapping from the surface to its associated K3 surface, but one can hope (and the Hodge conjecture would imply) that it is induced by a correspondence between the two surfaces. We use the term *isogeny* to denote a correspondence giving such an isomorphism. We have elsewhere [5, 7] given constructions of isogenies between algebraic surfaces with geometric genus one and their associated K3 surfaces in several particular cases.

If we require that the isomorphism between second transcendental cohomology groups preserve both the intersection pairings and the integral Hodge structures, then most algebraic surfaces with geometric genus one have a unique associated K3 surface (see Theorem 1 for the precise statement). However, even when the associated K3 surface is not unique, a theorem of Mukai [8] guarantees the existence of an isogeny between any pair of associated K3 surfaces; hence, any algebraic surface with geometric genus one has a unique isogeny class of associated $K3$ surfaces.

The "functoriality property" of our construction should be the follow-

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