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Reducibility of Flow-Spines

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The notion of flow-spines was introduced in [2]. A flow-spine is a standard spine of a closed 3-manifold M and is generated by a normal pair which is a pair of a non-singular flow on M and its compact local section. In this paper, we consider methods for constructing a simpler flow-spine than given one. In general, a spine P_1 (not necessarily a flow-spine) is thought to be simpler than P_2 when P_1 has less third singularities than P_2. And, for example in [1], several methods for obtaining a spine with less third singularities are discovered by Ikeda, Yamashita and Yokoyama. However a spine obtained by applying those methods to a flow-spine is not always a flow-spine. Hence, in order to leave our discussion within an extent of flow-spines, we must consider other "reducibility" of flow-spines.

In §4 we will give one of reasonable definitions of the reducibility of flow-spines. In §3 a "simply reduced flow-spine" is defined, and our reducibility will be considered within this sub-class of simply reduced flow-spines. And in §§5–6 we will give some conditions for a flow-spine to be reducible in our sense. §§1–2 are devoted to preparations. Especially in §2, we will precisely formulate the concept of a "singularity-data" introduced in [2], and give a necessary condition for a singularity-data to be realized by a normal pair.

§1. Preliminaries.

Let M be a smooth closed 3-manifold, and ψ_t be a smooth non-singular flow on M. A pair of ψ_t and its compact local section Σ is said to be a *normal pair* (see [2] for the precise definition), if (ψ_t, Σ) satisfies that

(i) Σ is homeomorphic to a compact 2-disk,

(ii) $|T_\pm(\psi_t, \Sigma)(x)| < \infty$ for any $x \in M$,

(iii) $\partial\Sigma$ is ψ_t-transversal at $(x, T_+(\psi_t, \Sigma)(x))$ for any $x \in \partial\Sigma$, and

(iv) if $x \in \partial\Sigma$ and $x_1 = \hat{T}_+(\psi_t, \Sigma)(x) \in \partial\Sigma$, then $\hat{T}_+(\psi_t, \Sigma)(x_1)$ is contained

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