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On Seminormal Underrings

Dedicated to Professor Masayoshi Nagata on his 60th birthday

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§1. Introduction and notation.

Let R be a (commutative integral) domain with quotient field K. One theme of enduring interest has been the study of R by analyzing properties of its overrings (that is, the rings contained between R and K). It seems remarkable that analogous "dual" studies have not been done in terms of the behavior of the underrings of R. (We shall say that B is an *underring* of R in case B is a subring of R also having quotient field K.) In [2], one took a first step by characterizing the R such that each *underring* of R is a Euclidean domain. These domains R were actually studied earlier by Gilmer [3] as the domains each of whose *subrings* is a Euclidean domain. In [1, Proposition 2.11], it was shown that the same domains R are characterized by requiring that each *subring* of R is seminormal. (As noted in [4, Theorem 1.1], a domain D, with quotient field L, is seminormal if and only if, whenever $u\in L$ satisfies $u^2\in D$ and $u^3\in D$, then $u\in D$.) One is naturally led to ask if the same domains R are characterized by requiring that each *underring* of R is seminormal. In [2], this was answered in the affirmative in the special case $R=K$. Our main result, Theorem 2.2, answers the general question in the affirmative. Its proof is independent of, and somewhat easier than, the work in [2].

R, K retain the above meanings throughout, all subrings contain the 1 of the larger ring, ch denotes characteristic, and $\boldsymbol{F}_p$ denotes the prime field of characteristic $p>0$. Any unexplained material is standard, as in [5].

§2. Results.

In any study of domains via behavior of their underrings, certain

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