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A Note on Test Sufficiency in Weakly Dominated Statistical Experiments

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Introduction

Let $\mathscr{E}=(X, \underline{A}, P)$ be a statistical experiment or simply an experiment, i.e., X be a set, $\underline{A}$ a σ-field of subsets of X and P a family of probability measures on $\underline{A}$. A set N is called P-null if $p(N)=0$ for all $p\in P$, and written $N=\varnothing\,[P]$. For A and B in $\underline{A}$, we write $A\subset B\,[P]$ if $A-B=\varnothing\,[P]$. A subfield $\underline{B}$ of $\underline{A}$ is called test sufficient if for any $\underline{A}$-measurable test function f, i.e., $0\leqq f\leqq 1$, there exists a $\underline{B}$-measurable test function g such that $\int f\,dp=\int g\,dp$ for all $p\in P$.

An experiment $\mathscr{E}$ is called weakly dominated if there exists a measure λ on $\underline{A}$ such that (a) for each p in P, there exists a density $dp/d\lambda$ and $P\equiv\lambda$, i.e., all the λ-null sets are P-null and vice versa, and (b) for every family $\{A_\gamma;\gamma\in\Gamma\}$ consisting of subsets which are σ-finite with respect to λ, there exists a set U called essential supremum, which satisfies (b-1) $U\in\underline{A}$, (b-2) $A_\gamma\subset U\,[\lambda]$ for all $\gamma\in\Gamma$ and (b-3) if $A\in\underline{A}$ and $A_\gamma\subset A\,[\lambda]$ for all $\gamma\in\Gamma$, then $U\subset A\,[\lambda]$.

An experiment $\mathscr{E}$ is called majorized if for each $p\in P$, there exists a set $S(p)\in\underline{A}$ called an $\mathscr{E}$-support of p, which satisfies
S-1. $p(S(p))=1$, and
S-2. $P\ll p$ on $S(p)$, i.e., if $N\in\underline{A}$, $N\subset S(p)$ and $p(N)=0$, then $N=\varnothing\,[P]$.

A weakly dominated experiment $\mathscr{E}$ is majorized since for each $p\in P$, $\{x\in X;(dp/d\lambda)(x)>0\}$ is an $\mathscr{E}$-support of p.

In a majorized experiment there exists a subclass $\underline{F}$ of $\underline{A}$ called a maximal decomposition, which satisfies
D-1. for each $F\in\underline{F}$, there exists $p\in P$ such that $p(F)>0$ and $F\subset S(p)\,[P]$,
D-2. for any distinct sets F and G in $\underline{F}$, $F\cap G=\varnothing\,[P]$,
D-3. each $p\in P$ is concentrated on a countable number of sets in F and
D-4. if $A\in\underline{A}$ and $A\cap F=\varnothing\,[P]$ for all $F\in\underline{F}$, then $A=\varnothing\,[P]$.

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