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On the Exponentially Bounded C-semigroups

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Introduction

In this paper we are concerned with exponentially bounded C-semigroups introduced by Davies and Pang [1].

Let X be a Banach space and let $C: X\to X$ be an injective bounded linear operator with dense range. A family $\{S(t): 0\leqq t<\infty\}$ of bounded linear operators from X into itself is called an *exponentially bounded C-semigroup* if

(0.1) $S(t+s)C=S(t)S(s)$ for $t, s\geqq 0$, and $S(0)=C$,

(0.2) for every $x\in X$, $S(t)x$ is continuous in $t\geqq 0$,

(0.3) there exist $M\geqq 0$ and $a\geqq 0$ such that $\|S(t)\|\leqq Me^{at}$ for $t\geqq 0$.

For every $t\geqq 0$, let $T(t)$ be the closed linear operator defined by $T(t)x=C^{-1}S(t)x$ for $x\in D(T(t))\equiv\{x\in X: S(t)x\in R(C)\}$. We define the operator G by

$$D(G)=\{x\in R(C): \lim_{t\to 0+}(T(t)x-x)/t \text{ exists}\} \quad\text{and}$$
$$Gx=\lim_{t\to 0+}(T(t)x-x)/t \qquad \text{for} \quad x\in D(G)\ . \tag{0.4}$$

For every $\lambda>a$, define the bounded linear operator $L_\lambda: X\to X$ by $L_\lambda x=\int_0^\infty e^{-\lambda t}S(t)xdt$ for $x\in X$. It is known that G is closable with dense domain (see [1]) and by [2, (2.3)]

$$(\lambda-\bar{G})L_\lambda x=Cx \qquad \text{for} \quad x\in X \quad\text{and}\quad \lambda>a\ ,$$
$$L_\lambda(\lambda-\bar{G})x=Cx \qquad \text{for} \quad x\in D(\bar{G}) \quad\text{and}\quad \lambda>a\ , \tag{0.5}$$

where $\bar{G}$ denotes the closure of G. $\bar{G}$ is called the *C-c.i.g.* (*C-complete infinitesimal generator*) of $\{S(t): t\geqq 0\}$.

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