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A Uniqueness Set for the Differential Operator $\Delta_z+\lambda^2$

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Introduction

We consider the Laplacian

$$\Delta_z=(\partial/\partial z_1)^2+(\partial/\partial z_2)^2+\cdots+(\partial/\partial z_{d+1})^2$$

in the complex $d+1$ space $\boldsymbol{C}^{d+1}$. Let

$$M=\{z=(z_1, z_2, \cdots, z_{d+1})\in \boldsymbol{C}^{d+1}; z\neq 0, z^2=0\}$$

be the complex cone defined by the quadratic equation

$$z^2=z_1^2+z_2^2+\cdots+z_{d+1}^2=0 .$$

Suppose λ is an arbitrary complex number. The first named author showed in [13] that the entire function f on $\boldsymbol{C}^{d+1}$ satisfying the differential equation

$$(0.1) \qquad (\Delta_z+\lambda^2)f=0$$

is completely defined by its restriction values on the complex subvariety M. In this sense, we call the cone M a uniqueness set for the differential operator $\Delta_z+\lambda^2$.

We shall show in this paper that this phenomenon occurs locally at the origin. More precisely, we shall prove a semi-local version using the Lie ball.

Let $\tilde{B}(r)$ be the Lie ball of radius r with center at the origin in $\boldsymbol{C}^{d+1}$ (see definition in §1). The space of holomorphic functions on $\tilde{B}(r)$ is denoted by $\mathcal{O}(\tilde{B}(r))$. We shall denote by $\mathcal{O}_\lambda(\tilde{B}(r))$ the subspace of $\mathcal{O}(\tilde{B}(r))$ defined by the differential equation (0.1). Remark that $\mathcal{O}_0(\tilde{B}(r))=\mathcal{O}_\Delta(\tilde{B}(r))$ in our notation in our previous paper [8], [10], etc., and that $\mathcal{O}_0(\tilde{B}(r))$ is the space of harmonic functions on the Lie ball $\tilde{B}(r)$.

Let us consider the space of functions on $M\cap\tilde{B}(r)$:

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