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Remarks on Perturbations of Function Algebras

Junzo WADA

Waseda University

Introduction

Let A be a function algebra on a compact Hausdorff space X. The purpose of this paper is to investigate small perturbations of the algebraic structure of A. In particular, we study the stability of direct sums of function algebras. K. Jarosz ([2], [3]) proved that if two function algebras A, B are both stable, then direct sum $A\oplus B$ of A and B is stable. In this note we deal with direct sums of function algebras $\{A_\lambda\}$ of infinitely many and give a condition under which the direct sum $\bigoplus_\lambda A_\lambda$ of $\{A_\lambda\}$ is stable (Theorem 1.2). Moreover it is shown that this condition is also a necessary one in order that $\bigoplus_\lambda A_\lambda$ is stable for $\{A_\lambda\}$ with some conditions (Theorem 1.1).

§1. Definitions and results.

For a function algebra A we write $\mathrm{Ch}\,A$ and ∂_A for the Choquet boundary and the Shilov boundary for A respectively. We consider a function algebra A as a closed subalgebra containing constant functions of the algebra $C(\partial_A)$ of all complex-valued continuous functions on ∂_A with the supremum norm. A closed subset F of ∂_A is called a *p-set* for A if for any open neighborhood U of F there is an $f\in A$ such that $f(s)=\|f\|=1$ $(s\in F)$ and $|f(s)|<1$ $(s\in\partial_A\setminus U)$ (cf. [1]).

Let A be a function algebra. By an *ε-perturbation* of A we mean any multiplication $\times$ defined on the Banach space A such that

$$\|f\times g-fg\|\leqq\varepsilon\|f\|\,\|g\| \qquad (f,\ g\in A)\ .$$

We call a function algebra A *stable* if there is an $\varepsilon>0$ such that for any ε-perturbation $\times$ of A algebras A and $(A,\times)$ are isomorphic. The stability is equivalent to the following: There is an $\varepsilon_1>0$ such that if T is any linear isomorphism from A onto a function algebra C with

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