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Appell's Hypergeometric Function F_2 and Periods of Certain Elliptic K3 Surfaces

Seiji NISHIYAMA

Tokyo Metropolitan University
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Introduction

In 1880 Appell introduced four types of hypergeometric functions F_1, F_2, F_3 and F_4 of two variables. These are generalizations of the Gauss hypergeometric function $F(\alpha, \beta, \gamma, x)$. There are several generalizations of the elliptic modular function $\lambda(\tau)$ or H. A. Schwarz's theory [14] using Appell's F_1 (see E. Picard [8, 9], T. Terada [17], P. Deligne and G. D. Mostow [2], H. Shiga [12, 13]). But there are no remarkable generalizations using F_2, F_3 and F_4.

In this paper we shall investigate an automorphic function of two variables derived from $F_2(\alpha, \beta, \beta', \gamma, \gamma', x, y)$ with $\alpha=\beta=\beta'=1/2$ and $\gamma=\gamma'=1$. To make the situation clear, let us recall what $\lambda(\tau)$ is. Consider the family $\mathscr{F}_0$ of the following elliptic curves $C(\lambda)$:

$$C(\lambda):\ w^2=u(u-1)(u-\lambda)\ ,\quad \lambda\in \boldsymbol{P}_1(\boldsymbol{C})-\{0, 1, \infty\}\ .$$

Let $\{\gamma_1, \gamma_2\}$ be a basis of $H_1(C(\lambda), \boldsymbol{Z})$ and assume that the intersection multiplicity $\gamma_1\cdot\gamma_2=-1$. And let ω be a holomorphic 1-form on $C(\lambda)$. Then the periods $\eta_i=\int_{\gamma_i}\omega$ $(i=1, 2)$ satisfy the following differential equation:

$$\lambda(1-\lambda)\frac{d^2z}{d\lambda^2}+(1-2\lambda)\frac{dz}{d\lambda}-\frac{1}{4}z=0\ .$$

This is the Gauss differential equation with $\alpha=\beta=1/2$ and $\gamma=1$. For the family $\mathscr{F}_0$, we define the period map τ on the parameter space $\boldsymbol{P}_1-\{0, 1, \infty\}$ by $\tau(\lambda)=\eta_1(\lambda)/\eta_2(\lambda)$. Then we have the following:

(1) *The image of τ is contained in upper half plane H.*

(2) *The inverse map $\lambda=\lambda(\tau)$ of τ is a single-valued holomorphic function on H mapped to $\boldsymbol{P}_1-\{0, 1, \infty\}$, and it is an automorphic function*

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