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Number Theoretical Transformations with Finite Range Structure and Their Ergodic Properties

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Introduction

Various number theoretical transformations, such as transformations for simple continued fraction [1], nearest integers continued fractions [10], complex continued fractions [2] [7], Jacobi-Perron's Algorithm [14], etc., have the following structure: Let $X\subset \boldsymbol{R}^n$ and T be a map of X onto itself. Then there exist a partition $\{X_a: a\in I\}$ of X, and a finite number of range sets $U_0, \cdots, U_N$ such that $T^n(X_{a_1}\cap T^{-1}X_{a_2}\cap\cdots\cap T^{-(n-1)}X_{a_n})\in \{U_i: 0\leqq i\leqq N\}$. The system $(X, T, \{U_0, \cdots, U_N\}, \{X_a; a\in I\})$ with such a structure will be called a *number theoretical transformation with finite range structure* (see the definition in §1).

In this paper, we first summarize ergodic properties of number theoretical transformations with finite range structure which have already been obtained in [10] and [8]. Namely, Theorem 1 in §1 states that the number theoretical transformation satisfying a transitivity condition and Renyi's condition is ergodic, exact, and admits a finite invariant measure whose density is bounded. Moreover, according to a result of Schweiger [15], Theorem 2 gives a sufficient condition in order that such a transformation possesses a σ-finite invariant measure. The maps defined by

$$T_1(\theta, \varphi)=\left(-\left[-\frac{1}{\theta}\right]-\frac{1}{\theta},\ -\left[-\frac{\varphi}{\theta}\right]-\frac{\varphi}{\theta}\right)$$

and

$$T_2(\theta, \varphi)=\left(-\left[-\frac{1}{\theta}\right]-\frac{1}{\theta},\ \frac{\varphi}{\theta}-\left[\frac{\varphi}{\theta}\right]\right)$$

are interesting examples in view of number theoretical applications [18] (cf. (3)).

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