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On the Asymptotic Behaviors of the Spectrum of Quasi-Elliptic Pseudodifferential Operators on R^n

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Introduction

We consider the asymptotic behaviors of the spectrum of pseudodifferential operators on $\boldsymbol{R}^n$ containing the Schrödinger operator:

$$(0.1)\qquad P(x, D)=-\Delta+V(x) \qquad \text{where} \quad \Delta=\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}\ .$$

If the potential $V(x)$ is a positive C^∞-function satisfying $\lim_{|x|\to\infty}V(x)=\infty$, then $P(x, D)$ is essentially self-adjoint in $L^2(\boldsymbol{R}^n)$ and its unique self-adjoint extension P is positively definite and has a compact resolvent in $L^2(\boldsymbol{R}^n)$. Therefore the spectrum of P consists only of eigenvalues of finite multiplicity: $\lambda_1\leqq\lambda_2\leqq\cdots$, $\lim_{k\to\infty}\lambda_k=+\infty$ with repetition according to multiplicity. Let $N_P(\lambda)$ be the counting function of eigenvalues: $N_P(\lambda)=\text{card}\{j;\lambda_j\leqq\lambda\}$.

In the particular case where $P(x, D)$ is the harmonic oscillator:

$$P(x, D)=-\Delta+V(x) \qquad \text{where} \quad V(x)=|x|^2\ ,$$

the asymptotic behavior of $N_P(\lambda)$ is well known (cf. Helffer and Robert [4]). Moreover Helffer and Robert [6] have obtained the asymptotic formula of $N_P(\lambda)$ for a class of quasi-elliptic pseudodifferential operators containing the anharmonic oscillator:

$$P(x, D)=-\Delta+V(x) \qquad \text{where} \quad V(x)=a|x|^{2k}\ (a \text{ real}>0,\ k \text{ integer}\geqq 2)\ .$$

They have found not only the first term but also the following several terms of $N_P(\lambda)$.

In this paper, we shall extend the result of [6] on $N_P(\lambda)$ for a class of quasi-elliptic pseudodifferential operators containing, in particular, the one on $\boldsymbol{R}^2$:

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