TOKYO J. MATH.
VOL. 10, NO. 2, 1987

Another Characterization of the Two-Weight Norm Inequalities for the Maximal Operators

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(Communicated by Y. Kawada)

Introduction

Let ν be a Borel measure on $\boldsymbol{R}^n$ and set

$$M_\alpha\nu(x)=\sup|Q|^{-\alpha}\int_Q d|\nu| \qquad (0<\alpha\leqq 1) \tag{0.1}$$

where the supremum is taken over all the cubes Q in $\boldsymbol{R}^n$ which contain x and $|Q|$ denotes the Lebesgue measure of Q. Throughout this note we deal with only cubes of the form $\prod_{j=1}^n [x_j, x_j+r)$ where $(x_1, \cdots, x_n)\in \boldsymbol{R}^n$ and $r>0$. $M_{1-\alpha/n}$, $0<\alpha<n$, is called the fractional maximal operator. When $\alpha=1$, (0.1) is the Hardy-Littlewood maximal function of ν.

Recently E. T. Sawyer [12] showed that for a nonnegative measure ω and a nonnegative function $v(x)$ on $\boldsymbol{R}^n$ there exists a positive constant C_1 independent of $f(x)$ such that

$$\left(\int_{R^n}[M_\alpha f]^q d\omega\right)^{1/q}\leqq C_1\left(\int_{R^n}|f|^p v dx\right)^{1/p} \qquad (0<\alpha\leqq 1) \tag{0.2}$$

for all measurable functions $f(x)$ if and only if there exists a positive constant C_2 independent of cubes Q such that

$$\int_Q [M_\alpha(\chi_Q v^{1-p'})]^q d\omega\leqq C_2\left(\int_Q v^{1-p'}\right)^{q/p}<\infty \tag{0.3}$$

for all cubes Q, where $1<p\leqq q<\infty$, $(1-p)(1-p')=1$ and χ_Q denotes the characteristic function of Q.

In the case that $p=q$, $\alpha=1$, ω is a function and $\omega=v$, as it is well known, B. Muckenhoupt [9] showed that (0.2) is valid if and only if ω satisfies A_p condition:

Received November 21, 1986
Revised March 12, 1987