TOKYO J. MATH.
VOL. 10, NO. 2, 1987

On a Degenerate Quasilinear Elliptic Equation with Mixed Boundary Conditions

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(Communicated by K. Kojima)

Introduction

Let Ω be a bounded simply connected domain in $\boldsymbol{R}^n$. The boundary $\partial\Omega$ is assumed to be of class C^1. Let S be a compact C^1 manifold of dimension $n-2$ belonging to $\partial\Omega$. We assume that S divide $\partial\Omega$ into two non-empty relatively open subsets $\partial_1\Omega$ and $\partial_2\Omega$, more precisely,

$$\partial\Omega=\partial_1\Omega\cup\partial_2\Omega\cup S\ , \qquad \partial_1\Omega\cap\partial_2\Omega=\varnothing\ .$$

We assume that the usual function spaces $C^k(\bar{\Omega})$, $C_0^k(\Omega)$, $L^q(\Omega)$, $W^{1,q}(\Omega)$, $W_0^{1,q}(\Omega)$ are known. The norm in $W^{1,q}(\Omega)$ $(L^q(\Omega))$ is written with $\|\ \|_{1,q}$ $(\|\ \|_q)$, respectively. Throughout this paper let $2<p<\infty$, and let all functions be real-valued. We set

$$C_{(0)}^1(\bar{\Omega})=\{u\in C^1(\bar{\Omega});\ u=0 \text{ in a neighborhood of } \overline{\partial_1\Omega}\}\ .$$

The completion of $C_{(0)}^1(\bar{\Omega})$ with respect to the norm $\|\ \|_{1,p}$ is denoted by $V(\Omega)$. The space $V(\Omega)$ is reflexive and separable. The norm in $V(\Omega)$ is denoted by $\|\ \|_V$. Let $V'(\Omega)$ be the dual space of $V(\Omega)$. As is well-known, Poincaré's inequality is valid for all functions in $V(\Omega)$, that is,

$$\|u\|_p\leqq C\|\nabla u\|_p\ , \qquad u\in V(\Omega)\ . \tag{0.1}$$

Hereafter let α be a real number such that

$$\begin{cases}\alpha\geqq 0 & \text{when} \quad p\geqq n\ ,\\ 0\leqq\alpha\leqq\dfrac{n(p-1)}{n-p}-1 & \text{when} \quad p<n\ .\end{cases} \tag{0.2}$$

We denote by $(\ ,\ )$ the inner product in $L^2(\Omega)$. For $u\in V(\Omega)$ we define

Received November 20, 1986
Revised August 1, 1987