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# On the Propagation of Chaos for Diffusion Processes with Drift Coefficients Not of Average Form

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## Introduction

Let $b^{(n)}[x, u]$ be $\boldsymbol{R}^d$-valued measurable functions defined on $\boldsymbol{R}^d \times \mathscr{P}(\boldsymbol{R}^d)$, where $\mathscr{P}(\boldsymbol{R}^d)$ denotes the space of probability distributions on $\boldsymbol{R}^d$. We consider interacting diffusion processes on $\boldsymbol{R}^d$ described by a system of stochastic differential equations

$$(1) \qquad X_i^{(n)}(t) = X_i^{(n)}(0) + B_i(t) + \int_0^t b^{(n)}[X_i^{(n)}(s), U^{(n)}(s)]ds , \qquad i=1, 2, \cdots, n ,$$

where $U^{(n)}(t) = (1/n) \sum_{i=1}^n \delta_{X_i^{(n)}(t)}$ is the empirical distribution of $(X_1^{(n)}(t), \cdots, X_n^{(n)}(t))$ and $B_i(t)$, $i=1, 2, \cdots, n$, are mutually independent $d$-dimensional Brownian motions. The initial value $(X_1^{(n)}(0), \cdots, X_n^{(n)}(0))$ is always assumed to be independent of the Brownian motions.

Assuming that the law of large numbers $U^{(n)}(\mathrm{t}) \rightarrow u(t)$ and $b^{(n)}[X_1^{(n)}(t), U^{(n)}(t)] \rightarrow b[X(t), u(t)]$ hold as $n \rightarrow \infty$ and taking the limit formally in (1) for $i=1$, we get the McKean-Vlasov's SDE

$$(2) \qquad X(t) = X(0) + B(t) + \int_0^t b[X(s), u(s)]ds ,$$

where $u(t)$ is the probability distribution of $X(t)$.

The propagation of chaos for the diffusion processes $(X_1^{(n)}(t), \cdots, X_n^{(n)}(t))$ given by (1) states as follows: If the sequence of the initial distributions in (1) is a symmetric $u$-chaotic family (see §2), then the sequence of the distributions of $(X_1^{(n)}(t), \cdots, X_n^{(n)}(t))$ is also a symmetric $u(t)$-chaotic family, where $u(t)$ is the probability distribution of $X(t)$ in (2) with a $u$-distributed initial value $X(0)$.

When the drift coefficient $b[x, u] = b^{(n)}[x, u]$, $n \geqq 1$, is of average (or integral) form defined by

$$(3) \qquad b[x, u] = \int b(x, y)u(dy) ,$$

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