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# The Universality of the Spaces of Ultradistributions $\mathscr{E}_s(T)^\wedge$, $\mathscr{E}_{(s)}(T)^\wedge$, $(0<s\leqq\infty)$, $\mathscr{E}_0(T)^\wedge$ and $\mathrm{Exp}(C^\times)^\wedge$

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## Introduction

L. Waelbroeck [11] proved the universality of the space $\mathscr{E}(V)^\wedge$ of Schwartz-distributions with compact support on a $C^\infty$-manifold $V$ with the $\delta$-function mapping $\delta: V\to\mathscr{E}(V)^\wedge$, i.e., any vector valued $C^\infty$-mapping $f: V\to E$ factors through $\delta: V\to\mathscr{E}(V)^\wedge$ by a uniquely determined linear morphism $f^\sim: \mathscr{E}(V)^\wedge\to E$ as $f=f^\sim\circ\delta$. For the unit circle $T$, we proved in a previous paper [9] that any vector valued $C^\omega$-mapping $f: T\to E$ factors through $\delta: T\to\mathscr{B}(T)$ where $\mathscr{B}(T)$ is the space of Sato-hyperfunctions on $T$. In the case of Schwartz-distributions, Waelbroeck used a notion of $b$-spaces, and for Sato-hyperfunctions we used a notion of $ib$-spaces (intersections of $b$-spaces).

In this paper we prove the universality for the spaces of ultradistributions of various kinds on the unit circle $T$. We represent these spaces as linear subspaces of $C^Z$ (called here sequence spaces) using Fourier coefficients. For a sequence space $E\subset C^Z$, Köthe [4] defined the dual (called $\alpha$-dual by him) by

$$E^\wedge=\{v\in C^Z \mid \text{for all } u\in E,\ \textstyle\sum_j |u_j|\,|v_j|<+\infty\}\ .$$

We consider functionals only of *sequential type*, i.e., $v: E\to C$ is represented as $v(u)=\langle u, v\rangle=\sum_j u_jv_j$ with $v\in E^\wedge$. A sequence space $E$ is *perfect* if $E=E^{\wedge\wedge}$. A linear mapping $f: E\to F$ between two sequence spaces is called *sequential* if for every $v\in F^\wedge$, the composed mapping $v\circ f\in E^\wedge$. If $E$ and $F$ are perfect, $f: E\to F$ is sequential if and only if $f$ is represented by an infinite matrix $(f_{ij})\in C^{Z\times Z}$ such that

$$\sum_j |f_{kj}|\,|u_j|<+\infty \qquad\text{and}\qquad \sum_k |v_k|\,\Big|\sum_j f_{kj}u_j\Big|<+\infty$$

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