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Stable Harmonic Maps from Riemann Surfaces to Compact Hermitian Symmetric Spaces

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Introduction

A harmonic map f from a compact Riemannian manifold N to a Riemannian manifold M is called *stable* if the second variation of the energy is nonnegative for every variation of the map f. A beautiful class of compact Kaehler manifolds is a class of irreducible Hermitian symmetric spaces of compact type ($P_m(C)$, $Q_m(C)$, $G_{p,q}(C)$, $Sp(k)/U(k)$, $SO(2k)/U(k)$, $E_6/Spin(10)\cdot T^1$, $E_7/E_6\cdot T^1$). We consider stable harmonic maps from or to compact Hermitian symmetric spaces. In this note we show the following.

THEOREM. *Let M be a compact irreducible Hermitian symmetric space of complex dimension m and Σ be a compact Riemann surface. Then any stable harmonic map f from Σ to M is holomorphic or anti-holomorphic.*

In the case where Σ is a Riemann sphere, the above result was obtained by Siu [S-1, Z] (see also [B-R-S]). In the case where M is a complex projective space and $|\deg f|\geqq m(p-1)/(m+1)$ where p denotes the genus of Σ, the above result was obtained by Eells and Wood [E-W]. They used algebraic geometric arguments (theorems of Riemann-Roch and Grothendieck). Recently, by using a twistor space over the domain manifold, Burns and de Bartolomeis have shown the above result in the case where M is a complex projective space (cf. Remark 6 of [B-R-S]). We show the above theorem by a simple argument for the second variations used in [L-S].

1. Proof of Theorem.

Let $f: N\to M$ be a harmonic map from an n-dimensional compact

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