TOKYO J. MATH.
VOL. 10, NO. 2, 1987

A Decomposition Theorem for Simple Lie Groups Associated with Parahermitian Symmetric Spaces

Soji KANEYUKI

Sophia University

Introduction

Let G/H be a semisimple affine symmetric space and let σ be the associated involutive automorphism of G. Let K be a σ-stable maximal compact subgroup of G. Then it is known (Flensted-Jensen [1], Rossmann [9]) that G admits the decomposition $G=KCH$ (with intersection), where C is a so-called split Cartan subgroup of G. In this paper we are mainly concerned with a simple parahermitian symmetric space M whose Weyl group $W(M)$ coincides with the Weyl group $W(M^*)$ of the fiber M^* of the Berger fibration of M ([5]). We then obtain a decomposition theorem for the simple Lie group G which arises as the automorphism group of M (Theorem 3.6). More precisely, we have the decomposition (with intersection) $G=KCH_l$ $(0\leqq l\leqq r=\dim C)$, where H_0 is the isotropy subgroup of G at a point in M, and H_l $(1\leqq l\leqq r)$ is the isotropy subgroup of G at a point on the boundary of M in a certain compactification of M. This is a partial generalization of the above-mentioned decomposition due to Flensted-Jensen and Rossmann; actually, when $l=0$, our decomposition is theirs. In Appendix, we give the table of the rank of the operator $\mathscr{K}(x)$ for each simple parahermitian symmetric space. That operator played an essential role in our previous paper [4].

§1. Basic facts.

Throughout this paper we shall use the terminologies in the previous papers [4], [2], [3]. Let $(\mathfrak{g}, \mathfrak{h}, \sigma)$ be a simple symmetric triple satisfying the condition:

(C) there exists an element $Z\in\mathfrak{g}$ such that $\operatorname{ad} Z$ is a semisimple operator with eigenvalues $0, \pm 1$ only and that $\mathfrak{h}$ is the centralizer of Z in $\mathfrak{g}$.

We denote by $\mathfrak{m}^{\pm}$ the eigenspaces in $\mathfrak{g}$ under the operator $\operatorname{ad} Z$, and put

Received December 10, 1986