An Extension of the Method of Iwahori Algebra

Dedicated to Professor Nagayoshi Iwahori on his 60th birthday

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Introduction

This paper is a study of three different types of induced representations of algebraic groups over an algebraically closed field K. As an application of it we can extend the method of Hecke algebras or Iwahori algebras of finite Chevalley groups introduced by N. Iwahori over Z, the ring of integers (see [4]) to the case of Chevalley groups G over K.

In section 1 we shall define the induced modules, but in case of Chevalley group G these three induced modules are given as follows. Let B be a certain Borel subgroup of G as in [10, §3] and $K^\times = K - \{0\}$. Let $\lambda: B \to K^\times$ be a rational linear character of B into $K^\times$. We shall write λ_B^G, $KG * \bar{\lambda}$ and $\mathrm{ind}_B^G \lambda$ respectively for the three induced modules induced from λ, where KG is the algebra of G over K.

DEFINITIONS.

$$\lambda_B^G = \{f: G \to K \mid f(bg) = \lambda(b)f(g) \text{ for any } b \in B \text{ and } g \in G\}$$

(see [5]). We define $g * f$, where $g \in G$ and $f \in \lambda_B^G$, to be the map of G into K which takes $x \in G$ to $f(xg)$, i.e.,

$$g * f(x) = f(xg) \qquad (x, g \in G).$$

$$\mathrm{ind}_B^G \lambda = \{f \in K[G] \mid f(bg) = \lambda(b)f(g) \text{ for any } b \in B \text{ and } g \in G\},$$

i.e.,

$$\mathrm{ind}_B^G \lambda = K[G] \cap \lambda_B^G, \text{ where } K[G] \text{ is the coordinate ring of } G.$$

We define $\bar{\lambda}$ to be the map of G into K which takes $x \in G - B$ to 0 and $x \in B$ to $\lambda(x)$, then $\bar{\lambda} \in \lambda_B^G$ and

$$KG \otimes_{KB} L \cong KG * \bar{\lambda} \qquad \text{(see Proposition 3.1)}$$