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On a p-Adic Interpolation of the Generalized Euler Numbers and Its Applications

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Introduction

The Euler numbers E_n are defined by

$$\frac{2}{e^t+1}=\sum_{n=0}^{\infty} E_n \frac{t^n}{n!} .$$

They are classical and important in number theory. Frobenius ([4]) extended E_n to the Euler numbers $H^n(u)$ belonging to an algebraic number u (see also §1), and many authors (e.g. [2], [4] and [8]) investigated their properties. Recently Shiratani-Yamamoto ([10]) constructed a p-adic interpolation $G_p(s, u)$ of the Euler numbers $H^n(u)$, and as its application, they obtained an explicit formula for $L_p'(0, \chi)$ with any Dirichlet character χ, including Ferrero-Greenberg's formula ([5]), and gave an explanation of Diamond's formula ([3]).

In the present paper, we shall define the generalized Euler numbers $H_\chi^n(u)$ for any Dirichlet character χ, which are analogous to the generalized Bernoulli numbers (see §1), and we shall construct their p-adic interpolation (see §2), which is an extension of Shiratani-Yamamoto's p-adic interpolation $G_p(s, u)$ of $H^n(u)$. The function $G_p(s, u)$ interpolates the n-th Euler number for $n\geqq 0$ with $(p-1)|n$, but our function interpolates the n-th generalized Euler number for any n. As applications, we shall obtain some congruences for the generalized Euler numbers (see §3), which improve the congruences for the Euler numbers in [2], [4] and [8]. In the last section, we shall define an element of a group ring. By using it, we shall reconstruct a p-adic interpolation of the Euler numbers in the Iwasawa method which makes use of the formal power series (cf. [12], §7.2).

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