TOKYO J. MATH.
VOL. 10, NO. 2, 1987

On Some Hypersurfaces of High-Dimensional Tori Related with the Riemann Zeta-Function

Kohji MATSUMOTO and Tetsuro MIYAZAKI

Rikkyo University and University of Tokyo
(Communicated by K. Katayama)

§1. Background of the problem.

Let $s=\sigma+it$ be a complex variable, and $\zeta(s)$ the Riemann zeta-function. Bohr-Jessen [1] [2] discussed the value-distribution of $\zeta(s)$ on the line $\sigma=\sigma_0$ ($>1/2$), and proved the following result:

Let R be any closed rectangle in the complex plane with the edges parallel to the axes, and $L(T)$ the measure of the set

$$\{t\in[0,\ T]\,|\log\zeta(\sigma_0+it)\in R\}\ .$$

Then, there exists the limit $W=\lim(L(T)/T)$ as T tends to infinity, which depends only on σ_0 and R. (In case $1/2<\sigma_0\leqq 1$, caused by the possibility of the existence of the zeros of ζ(s), there must be a slight modification.)

For proving this result, Bohr-Jessen introduced the set

$$\Omega(R)=\Omega_N(R)=\{(\theta_1,\ \cdots,\ \theta_N)\in[0,\ 1)^N\,|\,S_N(\theta_1,\ \cdots,\ \theta_N)\in R\}\ ,$$

where N is a large positive integer, and

$$S_N(\theta_1,\ \cdots,\ \theta_N)=-\sum_{n=1}^{N}\log(1-p_n^{-\sigma_0}\exp(2\pi i\theta_n))$$

(p_n denotes the n-th prime number). In fact they showed that, if we denote the measure of the set $\Omega_N(R)$ by W_N, then W_N tends to W, as N tends to infinity.

Recently, the first-named author has tried to refine Bohr-Jessen's argument, and obtained, in case $\sigma_0>1$, the asymptotic formula

$$L(T)=WT+O(T(\log\log(T))^{-(\sigma_0-1)/7+\varepsilon})$$

Received January 29, 1986
Revised February 4, 1987