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Holomorphic Functions on the Complex Sphere

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Introduction.

Let d be a positive integer and $d\geqq 2$ (see Remark in § 3 for $d=0, 1$). Let us denote by $\mathcal{O}(\boldsymbol{C}^{d+1})$ the space of entire functions on $\boldsymbol{C}^{d+1}$. Suppose λ is an arbitrary complex number and $\mathcal{O}_\lambda(\boldsymbol{C}^{d+1})=\{f\in\mathcal{O}(\boldsymbol{C}^{d+1});\ \Delta_z f=-\lambda^2 f\}$, where $\Delta_z=\sum_{j=1}^{d+1}(\partial/\partial z_j)^2$.

Let us consider the algebraic variety $M_\rho=\{z\in\boldsymbol{C}^{d+1};\ z^2=\rho^2\}$, where $z^2=\sum_{j=1}^{d+1}z_j^2$ and $\rho\in\boldsymbol{C}$. If $\rho=0$, M_0 is a complex cone and has a singularity at $z=0$. On the other hand if $\rho\neq 0$, M_ρ is a complex manifold and holomorphically diffeomorphic to the complex sphere $\tilde{S}=M_1=\{z\in\boldsymbol{C}^{d+1};\ z^2=1\}$ by the transformation $z\to\rho z$. The space $\mathcal{O}(M_\rho)$ of holomorphic functions on the analytic set M_ρ is equal to $\mathcal{O}(\boldsymbol{C}^{d+1})|_{M_\rho}$ by the Oka-Cartan Theorem B.

Our first main result is as follows:

THEOREM 1. *The restriction mapping $F\to F|_{M_\rho}$ is a linear topological isomorphism of $\mathcal{O}_\lambda(\boldsymbol{C}^{d+1})$ onto $\mathcal{O}(M_\rho)$ if*

$$(*)\qquad (\lambda\rho/2)^{-n-(d-1)/2}J_{n+(d-1)/2}(\lambda\rho)\neq 0$$

holds for $n=0, 1, 2, \cdots$, where J_ν is the Bessel function of order ν.

If $\rho=0$, the condition $(*)$ holds automatically and Theorem 1 was proved in [7] and [8]. The case $\rho\neq 0$ is proved in this paper (Theorem 2.1). Remark that Theorem 1 holds locally at the origin (see [9] for $\rho=0$ and Corollary 2.4 for $\rho\neq 0$). We may interpret Theorem 1 as saying that the set M_ρ is a uniqueness set for the differential operators $\Delta_z+\lambda^2$. We described in [10] a uniqueness set for more general linear partial differential operators of the second order with constant coefficients.

In [5] the space $\mathrm{Exp}(\tilde{S})$ was defined to be the restriction to $\tilde{S}$ of the space $\mathrm{Exp}(\boldsymbol{C}^{d+1})$ of entire functions of exponential type. But this defini-

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