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Bloch Constants and Bloch Minimal Surfaces

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§ 1. Introduction.

The n-th Bloch constant b_n ($n \geqq 2$) will be defined in terms of radii of certain disks on minimal surfaces in the Euclidean space $\boldsymbol{R}^n$. As will be seen, b_2 is the familiar one in the complex analysis. We shall prove that

$$b_n \geqq n^{-1/2} b_2 , \qquad n \geqq 3 . \tag{1.1}$$

Let each component x_j of a nonconstant map $x=(x_1, \cdots, x_n)$ from the disk $D=\{|w|<1\}$ in the complex plane $|w|<\infty$, $w=u+iv$, into the Euclidean space $\boldsymbol{R}^n$ ($n \geqq 2$) be harmonic in D. Then, the set S of all pairs $(w, x(w))$, $w \in D$, or simply, the map x itself, is called a minimal surface if

$$x_u x_v = 0 , \quad x_u x_u = x_v x_v \qquad \text{in} \quad D , \tag{1.2}$$

where

$$x_u = (x_{1u}, \cdots, x_{nu}) , \qquad x_v = (x_{1v}, \cdots, x_{nv})$$

are partial derivatives and the products are inner; S is the one-to-one image of D by x.

Henceforward, $x: D \to \boldsymbol{R}^n$ always means a minimal surface, and somewhat informally, we regard S as a subset of $\boldsymbol{R}^n$.

The surface S is endowed with the metric

$$d(x(w_1), x(w_2)) = \inf_{\gamma} \int_{\gamma} |x_u(w)|\,|dw| ,$$

where $x(w_j) \in S$, $j=1, 2$, $|x_u|=(x_u x_u)^{1/2}$ and γ ranges over all (rectifiable) curves connecting w_1 and w_2 in D. One can also consider this a new metric in D other than the Euclidean metric. Obviously, $|x(w_1)-x(w_2)| \leqq d(x(w_1), x(w_2))$; the left-hand side is the Euclidean metric in $\boldsymbol{R}^n$.

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