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On Compact Generalized Jordan Triple Systems of the Second Kind

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Dedicated to Professor Nagayoshi Iwahori on his sixtieth birthday

Introduction.

A finite dimensional graded Lie algebra $\mathscr{G}=\sum\mathscr{G}_k$ over a field F of characteristic zero is said to be of the ν-th kind, if $\mathscr{G}_{\pm k}=\{0\}$ for $k>\nu$. Let B: $(x, y, z)\mapsto(xyz)$ be a triple operation on a vector space U over F. The operation B is called a generalized Jordan triple system, if the equality $(uv(xyz))=((uvx)yz)-(x(vuy)z)+(xy(uvz))$ is valid for $u, v, x, y, z\in U$. If, in addition, the relation $(xyz)=(zyx)$ holds for $x, y, z\in U$, then B is said to be a Jordan triple system. Koecher [5] and Meyberg [7] studied interesting relationship between Jordan triple systems with nondegenerate trace forms and symmetric Lie algebras $(\mathscr{G}, \tau)$; here $\mathscr{G}$ is a semisimple graded Lie algebra of the 1st kind with $\mathscr{G}_0=[\mathscr{G}_{-1}, \mathscr{G}_1]$, and τ is a grade-reversing involution of $\mathscr{G}$. Our main concern is to generalize this connection to the case of generalized Jordan triple systems. It is known (Kantor [3]) that to a generalized Jordan triple system B on U there corresponds a graded Lie algebra $\mathscr{L}(B)=\sum U_i$ with $U_{-1}=U$. The triple system B is called of the ν-th kind, if the graded Lie algebra $\mathscr{L}(B)$ is of the ν-th kind. Under a certain condition (A) for B (cf. §1), $\mathscr{L}(B)$ admits a grade-reversing involution τ_B. The pair $(\mathscr{L}(B), \tau_B)$ is considered to be a generalization of the symmetric Lie algebra corresponding to a Jordan triple system. On the other hand, K. Yamaguti [8] introduced the bilinear forms γ_B for a wider class of triple systems. For a generalized Jordan triple system B, the form γ_B is symmetric, and, as is seen in the present paper, it plays the same role as the trace form for a Jordan triple system does. Now suppose B is of the 2nd kind. The first aim of this paper is to prove the following implications (Propositions 2.4, 2.5, 2.10 and Theorem 2.8):

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