TOKYO J. MATH.
VOL. 11, No. 1, 1988

Flow-Spines and Seifert Fibred Structure of 3-Manifolds

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The concept of a flow-spine of a closed 3-manifold M was introduced in [5]. In this paper, we shall give a sufficient condition for M represented by a flow-spine to be Seifert fibred (Theorem 1 in §2). In §3 the orbit manifold and exceptional fibres are completely determined by a DS-diagram which is induced by a flow-spine. We give an example in §4. And in §5, we study Seifert fibred submanifolds and embedded tori determined by a DS-diagram.

§1. A flow-spine and a DS-diagram with E-cycle.

A *normal pair* (ψ_t, Σ) on a closed 3-manifold M is a pair of a non-singular flow ψ_t on M and its compact local section Σ satisfying that

(i) Σ is homeomorphic to a compact 2-disk,

(ii) $T_-(x)=\sup\{t<0 \mid \psi_t(x)\in\Sigma\}$ and $T_+(x)=\inf\{t>0 \mid \psi_t(x)\in\Sigma\}$ are both finite for any $x\in M$, and

(iii) $\partial\Sigma$ is ψ_t-transversal at $(x, T_+(x))\in\partial\Sigma\times\boldsymbol{R}$ for any $x\in\partial\Sigma$,

(for the precise definition, see [5]). Flow-spines $P_\pm=P_\pm(\psi_t, \Sigma)$ are defined by

$$P_-=\Sigma\cup\{\psi_t(x) \mid x\in\partial\Sigma,\ T_-(x)\leqq\psi_t(x)\leqq 0\}\ ,$$
$$P_+=\Sigma\cup\{\psi_t(x) \mid x\in\partial\Sigma,\ 0\leqq\psi_t(x)\leqq T_+(x)\}\ ,$$

each of which forms a standard spine of M. It was shown in [5] that any closed 3-manifold admits a normal pair on it.

On the other hand, the notion of a *closed fake surface* and a *DS-diagram* was introduced in [2] and [3]. For a closed fake surface P, we denote by $\mathfrak{S}_j(P)$ $(j=1, 2, 3)$ the set of the j-th singularities of P (see [2] for the definition). Let P be a closed fake surface which admits a local homeomorphism $f\colon S^2\to P$ (S^2 is the 2-sphere) such that $\sharp f^{-1}(x)=j+1$ for $x\in\mathfrak{S}_j(P)$ $(j=1, 2, 3)$. Such an f is called an *identification map*. Then $G=f^{-1}(\mathfrak{S}_2(P))$ is a 3-regular graph on S^2. We call (S^2, P, G, f) a DS-

Received April 8, 1987