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The Involutions of Compact Symmetric Spaces

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Dedicated to Professor Ichiro Satake on his sixtieth birthday

Introduction.

The main purpose of this paper is to determine the fixed point sets of the involutions of the compact symmetric spaces. We will explain a few interesting applications to illustrate the significance of the results and the use of our geometric method.

The symmetric spaces M are defined with the point symmetries s_o at the points o in M. B. Y. Chen and the author made the local study of the fixed point sets $F(s_o, M)$, [CN-2]; the local structure of each connected component, M^+, of $F(s_o, M)$ and its "orthogonal complement", M^-, for the selected space (the adjoint space) in every local isomorphism class of the compact symmetric spaces. In this paper we will complete the global study first. We like to point out its theoretical interest. Given a symmetric space M, we have the set, PM, of the pairs (M^+, M^-) of two symmetric subspaces which is well defined with appropriate identification. Now two (compact and connected) symmetric spaces M, N are isomorphic if and only if PM is isomorphic with PN in the obvious sense. And a homomorphism of M into N gives rise to a homomorphism of PM into PN. Here a homomorphism of a symmetric space into another means a smooth map which commutes with every point symmetry, (1.2). A local version is found in [CN-2]. Since a homomorphism from a connected space is exactly a totally geodesic mapping, the fact above gives a necessary condition for existence of totally geodesic embeddings, for instance.

Our geometric method as opposed to heavier exploitation of root systems takes the knowledge of PM as the basic information. Indeed PM is closely related to M itself in terms of geometric structure. For example, M is orientable if and only if every M^+ has an even dimension; furthermore the Euler number χM of M is the sum of the Euler numbers

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