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A Characterization of the Poisson Kernel Associated with $SU(1, n)$

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§1. Introduction.

Let G be a connected non-compact semisimple Lie group with finite center and of real rank one. Let $G=KAN$ be an Iwasawa decomposition of G and M the centralizer of A in K. Then K is a maximal compact subgroup of G and $\dim A=1$; G/K is a non-compact Riemannian symmetric space of rank one and K/M the Furstenberg boundary. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{a}+\mathfrak{n}$ be the corresponding Iwasawa decomposition of the Lie algebra $\mathfrak{g}$ of G. For $g\in G$ let $H(g)$ denote the unique element in $\mathfrak{a}$ such that $g\in K\exp H(g)N$ and let $\rho(H)=(1/2)\mathrm{tr}(\mathrm{ad}(H)|_{\mathfrak{n}})$ for $H\in\mathfrak{a}$.

The Poisson kernel associated with G is the function on $G/K\times K/M$ defined by

$$P(gK, kM)=\exp(-2\rho H(g^{-1}k)) .$$

Let D be the Laplace-Beltrami operator on G/K (cf. [H1], p. 386). Then the function on G/K defined by $gK\mapsto P^{s}(gK, kM)$ ($kM\in K/M$ and $s\in \boldsymbol{C}$) is an eigenfunction of D with eigenvalue, say, λ_s. Now we shall consider the converse.

CONJECTURE. Let F be a real valued, C^2 function of G/K satisfying the following three conditions:

(1) $DF=0$,

(2) $D(F^2)=\lambda_2 F^2$,

(3) $F(eK)=1$.

Then there exists an element $kM\in K/M$ such that $F(gK)=P(gK, kM)$ for $gK\in G/K$.

When $G=SO_0(1, n)$, this conjecture was proved in [CET]. In this paper we shall give a proof for the case of $G=SU(1, n)$; in order to state our result more precisely, we shall give the explicit forms of the Poisson kernel and the Laplace-Beltrami operator associated with $G=SU(1, n)$.

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