TOKYO J. MATH.
VOL. 11, NO, 1, 1988

Asymptotic Distribution of Eigenvalues of Non-Symmetric Elliptic Operators

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(Communicated by K. Ogiue)

Introduction.

In this paper we study asymptotic formulas of distributions of eigenvalues of operators associated with strongly elliptic sesquilinear forms which have non-symmetric top terms.

For operators associated with symmetric forms, Maruo-Tanabe [6] and Tsujimoto [10, 11] gave remainder estimates depending upon the smoothness of the coefficients. Maruo [7] refined and extended the results of [6] to the forms which have symmetric top terms and non-symmetric lower terms. For differential operators, Robert [9] obtained the same results as [7].

As for operators associated with the forms with non-symmetric top terms, however, only the result by Watanabe [13] seems to have been given. He assumed C^h-smoothness for the coefficients of the top terms to clarify his intention to give the formula for eigenvalues which distribute in a sector of the complex plane.

The purpose of this paper is to establish an asymptotic formula with the optimal remainder estimate in the case of C^{1+h}-smoothness. In the symmetric case, our result coincides with Maruo's formula.

Now, we explain notations before stating our result. Let $\Omega\subset \boldsymbol{R}^n$ $(n\geqq 2)$ be a bounded domain possessing the restricted cone property (see [1, p. 11]). Set $\Omega_\varepsilon=\{x\in\Omega;\ \delta(x)\geqq\varepsilon\}$ with $\varepsilon>0$ where $\delta(x)=\min\{1, \mathrm{dist}(x, \partial\Omega)\}$ for $x\in\Omega$. We impose on Ω the following condition: There exists a constant $C>0$ such that for any $\varepsilon>0$

(0.1) $$\int_{\Omega_\varepsilon}\delta^{-1}(x)dx<C|\log\varepsilon|\ ;$$

Received September 8, 1986
Revised August 28, 1987